Radiative effect on conjugate forced convection and conductive heat transfer in a circular pin

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An analysis is presented for the heat transfer characteristics of a laminar forced convective flow over a circular pin by the conjugate convection-conduction theory including radiative effects under optically thick limit approximation. Numerical results are presented for the dimensionless heat transfer coefficients, local and overall heat fluxes, and temperature distribution of the pin by a simultaneous solution of the convective boundary layer equations of the fluid and the energy equation of the pin. It is found that the radiative effect tends to increase the surface temperature and has the effect of augmenting the overall heat transfer rate of the pin.

Keywords: conjugate heat transfer; radiative heat transfer; extended surface heat transfer

Introduction

The problem of conjugate convection and conduction for a vertical plate fin has been recently studied by Sparrow and coworkers.[1,2] They argued that the conventional fin model based upon a uniform heat transfer coefficient cannot yield accurate results in the prediction of local heat fluxes. They therefore simultaneously solved the conduction problem for the fin and the convective heat transfer problem for the flowing fluid, and showed that the heat transfer coefficient may experience substantial variations along the fin surface.

In the present analysis, we are concerned with a circular pin fin extending from a wall and transferring heat to a surrounding fluid by forced convection and radiation. The temperature of the fin is not known a priori. The heat transfer coefficient along the circular pin is not prescribed, but will be determined from a solution of the boundary layer equations and its interaction with the pin conduction and radiation.

Analysis

Let us consider a uniform free stream with velocity U_∞, temperature T_∞, kinematic viscosity ν, and thermal conductivity k_f approaching a circular pin of radius R and length L. The pin is attached to a wall of temperature T_0. The thermal conductivity of the pin is k_w. The temperature of the pin T_w varies along its length. The axial and radial coordinates are taken to be x and r, respectively. The conservation equations for a laminar boundary layer over the circular pin may be written as

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial(r\,\partial u/\partial r)}{\partial r} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)-\frac{1}{\rho CP}\frac{1}{r}\frac{\partial}{\partial r}(rq^{r}) \tag{3}$$

The boundary conditions are

$$\begin{aligned} &r=R: && u=v=0, \quad T=T_w(x)\\ &r\to\infty: && u\to U_\infty, \quad T\to T_\infty\\ &x=0,\ r\geqslant R; && u=U_\infty, \quad T=T_\infty \end{aligned} \tag{4}$$

Proceeding with the analysis, we now introduce pseudosimilarity variables (ξ,η) with a reduced stream function $f(\eta)$ and a dimensionless temperature $\theta(\xi,\eta)$ as follows:

$$\begin{aligned} \xi&=\frac{x}{L}\\ \eta&=\frac{r^2-R^2}{4RL}\left(\frac{\mathrm{Re}}{\xi}\right)^{1/2}\\ f(\xi,\eta)&=\frac{\psi(x,r)}{R(\nu U_\infty x)^{1/2}}\\ \theta(\xi,\eta)&=\frac{T(x,r)-T_\infty}{T_0-T_\infty} \end{aligned} \tag{5}$$

The stream function $\psi(x,r)$ satisfies the continuity equation 1 automatically with

$$ru=\frac{\partial\psi}{\partial r}, \qquad rv=-\frac{\partial\psi}{\partial x} \tag{6}$$

We assume optically thick approximation for the radiative heat flux q^{r}. Upon substituting the expressions in Equation 5 into the governing momentum and energy equations, we may write the transformed equations as

$$(1+\xi^{1/2}\lambda\eta)f'''+(f+\xi^{1/2}\lambda)f''=2\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{7}$$

$$\begin{aligned} &\frac{1}{\mathrm{Pr}}(1+\xi^{1/2}\lambda\eta)\theta''+\left(f+\frac{\xi^{1/2}\lambda}{\mathrm{Pr}}\right)\theta'\\ &\qquad+\frac{4}{3\mathrm{Pr}\cdot N}[(1+\xi^{1/2}\lambda\eta)(\theta+C_T)^3\theta']'\\ &=2\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \end{aligned} \tag{8}$$

The transformed boundary conditions are given by

$$f(\xi,0)=f'(\xi,0)=0,\quad \theta(\xi,0)=\theta_w(\xi)$$
$$f'(\xi,\infty)=2,\quad \theta(\xi,\infty)=0 \tag{9}$$

In the above equations, primes denote partial differentiation with respect to η alone.

The length L of the pin is assumed to be longer compared to its diameter, and the pin temperature variations in the longitudinal direction may be considered to be larger than the transverse direction variations. Thus, the heat conduction along the pin is one-dimensional. The coupling between the pin and the convective flow is expressed by the requirement that the pin and fluid temperatures and heat fluxes be continuous at the pin-fluid interface at all x locations. This requirement is expressed as

$$T_w=T_f \quad \text{and} \quad -k_f\left(\frac{\partial T}{\partial r}\right)+q^r=h(T_w-T_\infty)$$

for $r=R$ and $0\leqslant x\leqslant L$ (10)

The pin conservation of energy equation is

$$\frac{\partial^2\theta_w}{\partial\xi^2}=Nc\cdot\bar{h}\cdot\theta_w \tag{11}$$

In the above equation,

$$\theta_w(\xi)=\frac{T_w(x)-T_\infty}{T_0-T_\infty}$$

$$\bar{h}=-\frac{\theta'(\xi,0)[1+4\{\theta(\xi,0)+C_T\}^3/3N]}{2\xi^{1/2}\theta(\xi,0)}$$

$$Nc=\frac{2Lk_f\,\mathrm{Re}^{1/2}}{k_wR} \tag{12}$$

The boundary conditions for Equation 11 are

$$\xi=0:\ \frac{\partial\theta_w}{\partial\xi}=0 \qquad \xi=1:\ \theta_w=1 \tag{13}$$

Numerical solution and discussion of results

The iterative procedure used for the circular pin and the boundary layer may be outlined as follows:

(1) Solve the convective boundary layer problem for the circular pin.
(2) Assume a temperature distribution that satisfies the thermal boundary conditions for $0\leqslant\xi\leqslant1$.
(3) Compute $\bar{h}$ from Equation 12.
(4) Solve Equation 11 for $\theta_w(\xi)$ using the $\bar{h}$ obtained above.
(5) Repeat the procedure of alternately solving the boundary layer problem and fin conduction problem until convergence is reached.

The boundary layer equations were solved by an implicit finite difference method described by Cebeci and Bradshaw.[3] The partial differential equations 7 and 8 were converted to a system of first-order equations, which were then written in finite difference form. The functions and their derivatives were approximated in centered difference and averages at midpoints of the net segments in the (ξ,η) coordinates. The resulting nonlinear finite difference equations were solved by Newton's iterative method. The pin conduction equation was expressed in finite difference form. For small ξ, a finer subdivision was used for the convective problem as well as the conduction problem.

The dimensionless local heat flux may be written as

$$\frac{q_wL}{k_f(T_0-T_\infty)\,\mathrm{Re}^{1/2}}=-\frac{\theta'(\xi,0)[1+4(\theta+C_T)^3/3N]}{2\xi^{1/2}} \tag{14}$$

The overall heat transfer rate Q_w from the pin may be obtained from the heat conducted from the wall into the pin base at $\xi=1$ or by integrating the local heat flux at the fin surface. We may write

$$\frac{Q_w}{k_f(T_0-T_\infty)\,\mathrm{Re}^{1/2}}=\frac{2}{Nc}\left(\frac{\partial\theta_w}{\partial\xi}\right)_{\xi=1} \tag{15}$$

The numerical results for the local heat flux at various axial locations of the pin are shown in Figure 1. Here, we have chosen the conduction-radiation parameter $N=1$, the temperature difference parameter $C_T=0.5$, Prandtl number $\mathrm{Pr}=0.7$, and the transverse curvature parameter $\lambda=1.0$. The convection-conduction parameter Nc was chosen as a parameter. It may be seen that the heat transfer rate of the pin with radiative effect is higher than in the absence of radiation. As Nc increases, we see that the heat flux decreases.

In Figure 2, we have presented the results for the overall heat transfer rate Q_w from the pin for several values of the

Notation

C_P	Specific heat
C_T	Temperature ratio $T_\infty/(T_0-T_\infty)$
f	Dimensionless stream function
h	Heat transfer coefficient
$\bar{h}$	Dimensionless heat transfer coefficient
k	Thermal conductivity
L	Length of the pin
N	Conduction-radiation parameter
Nc	Convection-conduction parameter
q	Local heat flux
q^r	Radiative heat flux $(-4\sigma/3\beta^*)\,\partial T^4/\partial y$ for optically thick limit
Re	Reynolds number $(U_\infty L/\nu)$
r	Radial coordinate
R	Radius of the pin
T	Temperature
U_∞	Free-stream velocity
u, v	Velocity components in x and r directions, respectively
x	Streamwise coordinate
α	Thermal diffusivity
β^*	Extinction coefficient
η, ξ	Pseudosimilarity variables
θ	Dimensionless temperature
λ	Transverse curvature parameter $(4L/R\cdot\mathrm{Re}^{1/2})$
ν	Kinematic viscosity
ρ	Density of fluid
σ	Stefan-Boltzmann constant
ψ	Stream function

Subscripts

f	Properties of the fluid
w	Properties of the solid pin
0	Conditions at root of the pin
∞	Conditions at free stream

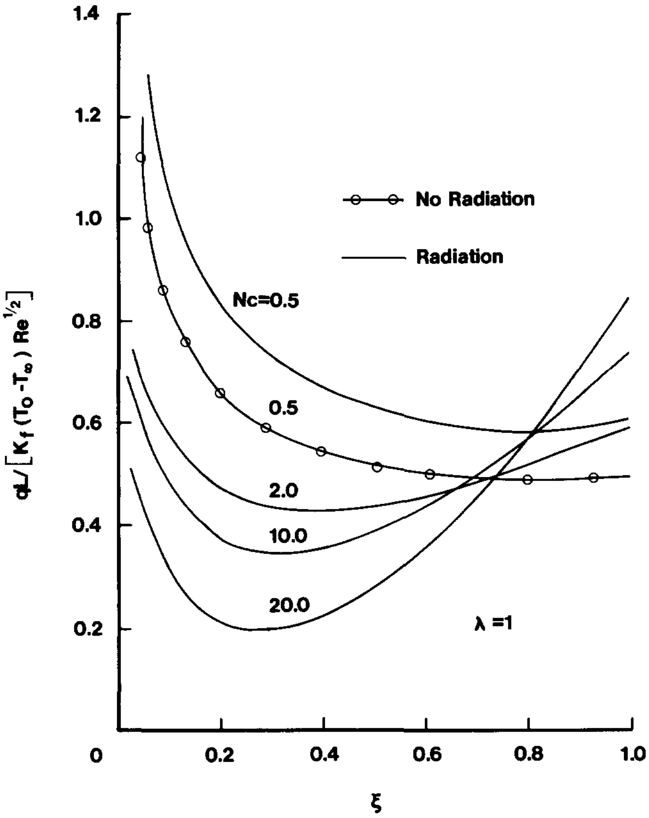

Figure 1 Local heat flux distribution along the pin surface (N=1, C_T=0.5, Pr=0.7)

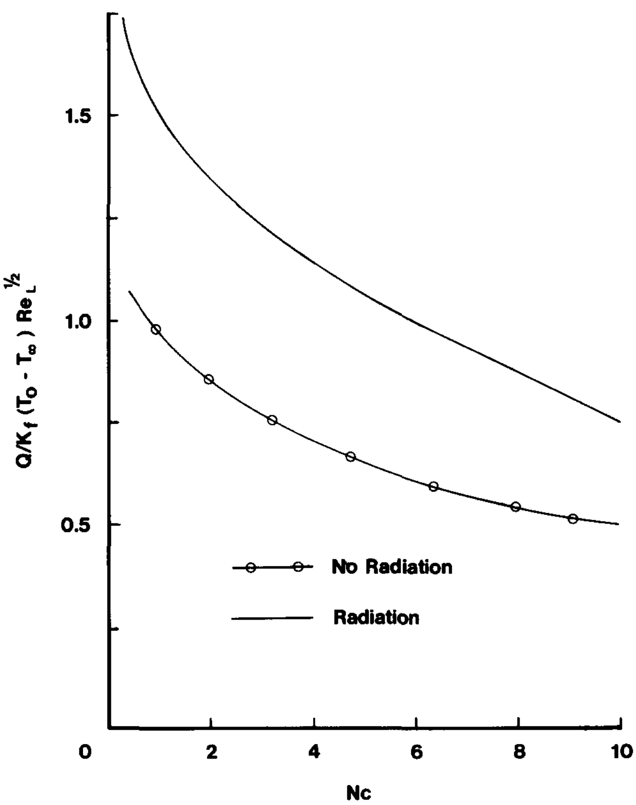

Figure 2 Overall heat flux rate variation

convection-conduction parameter Nc. The results indicate that the overall heat transfer rate increases due to the radiative effect.

The distributions of the dimensionless local heat transfer coefficients along the pin surface are shown in Figure 3. In the

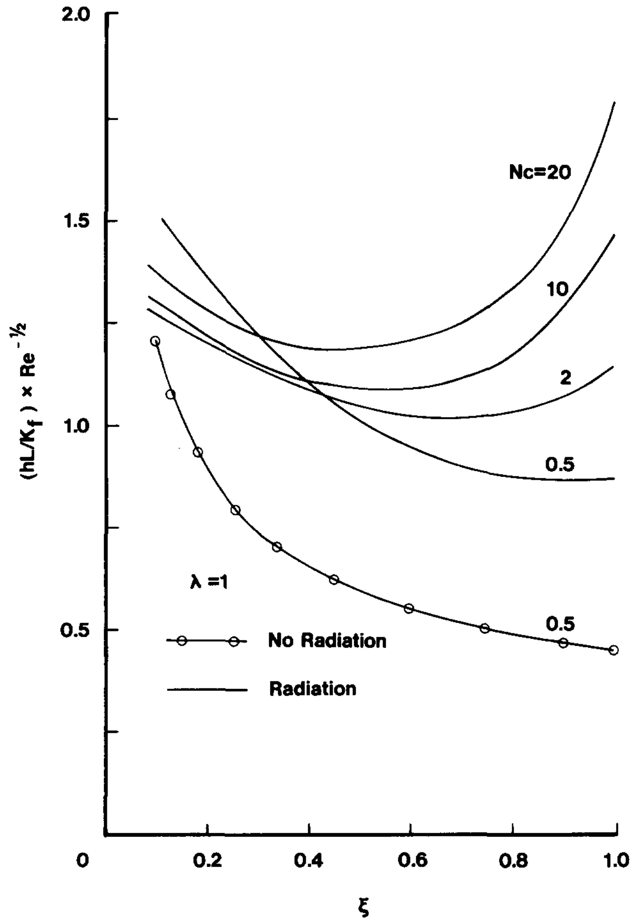

Figure 3 Distribution of local heat transfer coefficients along the pin surface

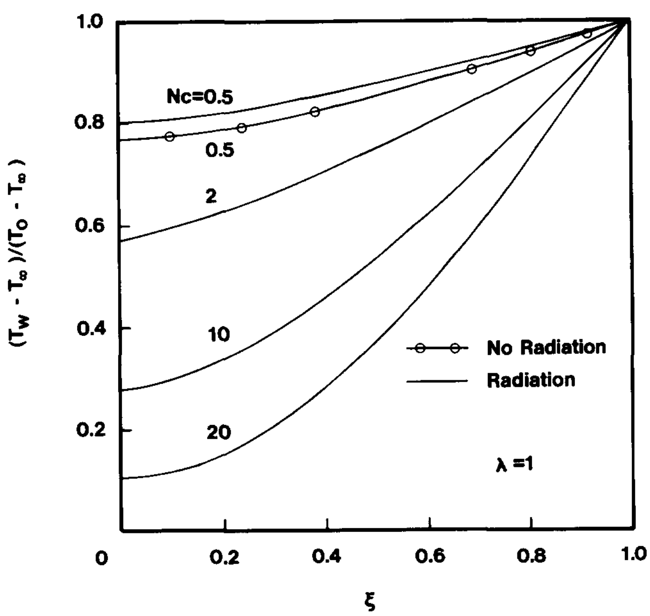

Figure 4 Distribution of temperature of the pin

absence of radiative effects, the heat transfer coefficient tends to decrease monotonically in the fluid flow direction. However, when radiative effects are included, we notice that the dimensionless heat transfer coefficients do not decrease monotonically. For high values of N, the heat transfer coefficients decrease initially and then increase. The effect of including radiation is to augment the heat transfer coefficients.

The results for the pin temperature distribution are shown in Figure 4. The pin temperature decreases monotonically from root to tip. The effect of radiation is to increase the surface temperature of the pin for a fixed value of Nc. As Nc increases, we see that the pin surface becomes more nonisothermal.

Concluding remarks

In this paper, we have presented an analysis for the conjugate convection and conduction heat transfer with radiative effects under the optically thick limit approximation for forced flow over a circular pin. The overall heat transfer rate decreases with increasing values of the convection-conduction parameter Nc. The radiative effect augments the overall heat transfer rate. The surface temperature variations of the pin from the root to tip increase with increasing values of Nc or decreasing values of thermal conductivity of the pin, k_w. The radiative effect, in general, tends to increase the surface temperature of the pin.

References

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